5-7 Videos Guide

5-7a

- Surface integral of a scalar field $f(x, y, z)$
  - $\iint_S f(x, y, z)\, dS = \iint_D f(\mathbf{r}(u, v))|\mathbf{r}_u \times \mathbf{r}_v|\, dA$
    Note that $dS = |\mathbf{r}_u \times \mathbf{r}_v|\, dA$.

Exercises:

- Evaluate the surface integral.
  - $\iint_S xyz\, dS$,
    S is the cone with parametric equations $x = u \cos v$, $y = u \sin v$, $z = u$, $0 \le u \le 1$, $0 \le v \le \pi/2$

5-7b

  - $\iint_S xy\, dS$,
    S is the part of the plane $2x + 2y + z = 4$ that lies in the first octant

5-7c

- If S consists of multiple surfaces S_i, then
  $\iint_S f(x, y, z)\, dS = \iint_{S_1} f(x, y, z)\, dS + \iint_{S_2} f(x, y, z)\, dS + \cdots \iint_{S_n} f(x, y, z)\, dS$

Exercise:

- Evaluate the surface integral.
  $\iint_S (x^2 + y^2 + z^2)\, dS$,
  S is the part of the cylinder $x^2 + y^2 = 9$ between the planes $z = 0$ and $z = 2$, together with its top and bottom disks

5-7d

- Surface integral of a vector field $\mathbf{F}(x, y, z)$
  - Flux is $\iint_S \mathbf{F} \cdot d\mathbf{S} = \iint_S \mathbf{F} \cdot \mathbf{n}\, dS = \iint_D \mathbf{F} \cdot (\mathbf{r}_u \times \mathbf{r}_v) dA$
    Note that $d\mathbf{S} = \mathbf{n} dS = (\mathbf{r}_u \times \mathbf{r}_v)\, dA$, where $\mathbf{n}$ is a unit normal vector and $\mathbf{r}_u \times \mathbf{r}_v$ is simply a normal vector to the surface S.
  - If x and y are the parameters, we have
    $\iint_S \mathbf{F} \cdot d\mathbf{S} = \iint_D \left(-P \frac{\partial g}{\partial x} - Q \frac{\partial g}{\partial y} + R\right) dA$, for upward orientation. The signs of the integrand change for downward orientation.

Exercises:

5-7e

- Evaluate the surface integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$ for the given vector field $\mathbf{F}$ and the oriented surface S. In other words, find the flux of $\mathbf{F}$ across S. For closed surfaces, use the positive (outward) orientation.
    - $\mathbf{F}(x, y, z) = -x\,\mathbf{i} - y\,\mathbf{j} + z^3\,\mathbf{k}$,
      S is the part of the cone $z = \sqrt{x^2 + y^2}$ between the planes $z = 1$ and $z = 3$ with downward orientation

5-7f

- $\mathbf{F}(x, y, z) = x\,\mathbf{i} + y\,\mathbf{j} + 5\,\mathbf{k}$,
  S is the boundary of the region enclosed by the cylinder $x^2 + z^2 = 1$ and the planes $y = 0$ and $x + y = 2$